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MATRIX MODELS, OPEN STRINGS AND QUANTIZATION OF MEMBRANES[1]

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ABSTRACT

We present an approach to membrane quantization using matrix quantum mechanics at large N. We show that this leads (through a simple field theory of two-dimensional open strings and the associated $SU(\infty)$ current algebra) to a 4-D dynamics of self-dual gravity plus matter.

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The problem of quantizing relativistic membranes (and higher dimensional p-branes) is of major relevance. These objects are seen to appear as extended solitonic states of ten-dimensional superstring theory [1, 2, 3, 4] and participate in the corresponding weak-strong coupling duality maps. Even more, the membrane is expected to play a fundamental role as the basic object of the 11(12) dimensional M (F) theory [5, 6, 7] .

A fruitful idea for constructing the world volume description of extended p-branes is given by the notion of world sheet - target space duality [8, 9]. In this, a target space field theory of lower dimensional branes is conjectured to give a **world volume** description of a higher dimensional brane. In particular, a quantum theory of a membrane could arise as a field theory of 2-d strings.

Matrix models have given certain insight into a field theory of lower dimensional non-critical strings. We can then contemplate an approach to membrane quantization following the development of string equations and low dimensional string field theory. Indeed it was shown [10, 11] that a quantum membrane can be represented by a matrix model corresponding to a dimensional reduction of an $SU(N)$ (super) Yang-Mills theory. In the large N limit ($N \to \infty$) the $U(N)$ gauge group becomes a symmetry of area preserving diffeomorphisms representing a symmetry of the membrane Hilbert space. The nontrivial feature in the quantization of the membrane is connected with the treatment of this symmetry.

In what follows, we will describe an approach to this problem. We will begin with the matrix model formulation and develop a continuum field theory operating in the invariant subspace. The field theory involves a current algebra of $SU(R)$ type representing open string fields which appear as an intermediate construct [12] in this approach. The nontrivial matrix dynamics is thereby represented in terms of conformal fields. The $R \to \infty$ limit is then argued to be the theory of a membrane. In particular, we show that in this limit the dynamics is described by a 4-d **self-dual** gravitational field. The quantum Hamiltonian for this self-dual theory is then specified by the $SU(\infty)$ current algebra model.

To begin, it is a result of Ref. [11] that a relativistic (super-) membrane in the light cone gauge

$$X^{\pm} = X^{D} \pm X^{0}$$

reduces to a Hamiltonian for the transverse degrees of freedom

$$X_i\ (\sigma_1, \sigma_2)\ , \quad i = 1, 2, \cdots, D-2; \qquad \bar{\Theta}\ , \Theta^1\ (\sigma_1, \sigma_2)$$

taking the form

$$H = \int d^2\sigma \left(\frac{1}{2} \sum_{i=1}^{D-2} \dot{X}_i^2 + \frac{1}{2} \sum_{i<j} \{X_i, X_j\}^2 + \frac{i}{2} \bar{\Theta}\, \gamma^i \{X_i, \Theta\} \right)$$

with the constraint

$$G\,(\sigma_1, \sigma_2) = \sum_{i}^{D-2} \left\{ X_i, \dot{X}_i \right\} + \bar{\Theta}\Theta$$

representing area-preserving diffeomorphisms. The bracket

$$\{A, B\} = \epsilon^{rs}\, \partial_r\, A\, \partial_s\, B$$

corresponds to a large N limit of the matrix commutator and the correspondence with matrix notation is given through

$$X_i\,(t, \sigma_1, \sigma_2) = \lim_{N \to \infty} M^{ab}(t).$$

The matrix Hamiltonian

$$H = Tr \left\{ \frac{1}{2} \sum_{i=1}^{D-2} \dot{M}_i(t)^2 + \Sigma\,[M_i, M_j]^2 + \frac{i}{2} \bar{\Theta}\, \gamma^i\, [X_i, \Theta] \right\}$$

represents a dimensionally reduced SUSY Yang-Mills theory with only a time dimension preserved.

Consider in what follows the case of a 4-dimensional membrane ($D-2 = 2$), concentrating on the bosonic coordinates (we will simply comment on the supersymmetric extension at the appropriate place). The problem that we have is that of a two-matrix system at $N \to \infty$. Using ideas of collective field theory, we represent this in terms of **conformal fields** as follows: The nontriviality of the problem lies in the Gauss Law constraint:

$$G = \left[M_1, \dot{M}_1 \right] + \left[M_2, \dot{M}_2 \right] \approx 0.$$

Gauge symmetry

$$M_i(t) \to V^+(t)\, M_i\, V(t)$$

allows one to diagonalize one of the matrices, for example M_1:

$$M_1(t) = \mathrm{Diag}\ \left(\lambda_i(t)\right),$$

and then

$$P_1 = \dot{M}_1 \to\ p_i + \left[\dot{V}V^+, \lambda\right],$$

where

$$p_i = \dot{\lambda}_i \qquad 0 = 1, 2, \cdots, N.$$

The Gauss law constraint is then solved for the angle variables $\dot{V}V^+$ giving

$$P_1 = p_i\, \delta_{ij} + \frac{Q_{ij}}{(\lambda_i - \lambda_j)},$$

with $Q_{ij} = [M_2, \dot{M}_2]$ being the $SU(N)$ charge of the second matrix. We now introduce a **reduction**:

$$Q_{ij} = \sum_{a=1}^{R} \psi_i^+\,(a)\ \psi_j(a),$$

where $\psi_i(a)$ represent quark degrees of freedom with $a = 1, \ldots, R$ representing flavor indices. We expect that in the limit $R \to \infty$ the elements of the original matrix dynamics are recovered. The Hamiltonian problem at hand is then

$$H = \sum_{i=1}^{N} \frac{1}{2}\, \dot{\lambda}_i^2\ + \frac{1}{2} \sum_{i<j}\ \frac{(\psi^{+a}\psi^b)_i(\psi^{+b}\psi^a)_j}{(\lambda_i - \lambda_j)^2}\ + V.$$

This is a form of the dynamical spin Calogero problem. Here the operators

$$\left(\psi^{+a}\, \psi^b\right)_i$$

create open strings with $a, b = 1, \cdots, R$ representing the Chan-Paton factors of the $SU(R)$ group. This system has a continuum representation in terms of a field theory of open strings (formulated in collaboration with J. Avan and J. Lee). The collective fields are introduced

$$\sum_i^N \delta\ (x - \lambda_i(t)) \to \alpha_+(x,t)\ - \alpha_-(x,t)\,,$$

$$\sum_i \left(\psi^{+a}\psi^b\right)_i \delta\,(x - \lambda_i) \to\ J_+^{ab}\ \Big(x,t) - J_-^{ab}\,(x,t)\Big)\,,$$

giving variables of a $U(1)$ and $SU(R)$ affine current algebras. For the compact case one has $x \to z = e^{i\varphi}$, which corresponds to a unitary matrix $U = e^{iM}$. The continuum Hamiltonian describing the dynamics is

$$\begin{aligned} H \ &= \ \int dx \left\{\frac{1}{6}\left(\alpha_+^3 - \alpha_-^3\right) + \alpha_+ T(J_+) - \alpha_- T(J_-)\right\} \\ &+ \ c\sum_{\epsilon=\pm} \frac{J^{ab}_{(x)}\, J^{ba}_{(y)}}{(x-y)^2} + \gamma'\,(W_0(J_+) - W_0(J_-))\,. \end{aligned} \tag{1}$$

Here

$$T(J) = \frac{1}{R+1}\,\left(J^{ab}(x)\,J^{ba}(x)\right)$$

is the energy-momentum tensor (of the current algebra) and W_0 are the zero modes corresponding to the spin-3 W_3 generators.

For details of the theory and a discussion of relevant symmetry properties of the Hamiltonian the reader should consult refs. [12].

We have expressed the dynamics of the matrix model in terms of a $U(1)$ boson:

$$[\alpha_\pm(x), \alpha_\pm(y)] = \pm 2\,\partial_x\,\delta(x-y)$$

and $L - R$ $SU(R)$ affine currents

$$\begin{aligned} \left[J^\alpha_\pm(x), J^\beta_\pm(y)\right] \ &= \ f_{\alpha\beta\gamma} J^\gamma_\pm\,\delta(x-y) \pm \frac{\kappa}{2\pi}\,\delta^{\alpha\beta}\delta', \\ \left[J^\alpha_+, J^\beta_-\right] \ &= \ 0, \end{aligned} \tag{2}$$

representing degrees of freedom of a WZW model. Let us now consider the limit $R \to \infty$ and exhibit the associated continuum degrees of freedom that emerge. In this limit, our currents take the form

$$J^{ab}_\pm(t,x) \to J_\pm\,(t,x,\sigma_1,\sigma_2)\,,$$

thereby becoming a 4-dimensional field. The scalar field $\alpha_{\pm}$ obviously plays the role of a **dilaton**. Taking $\alpha_{\pm} = \pm\alpha_0$, the quadratic term of the Hamiltonian reads

$$H_2 = \int dx\, \alpha_0 \left(J_+^2 + J_-^2 \right),$$

representing a Hamiltonian of an $SU(\infty)$ σ-model. We now use arguments due to Park [13] to show that the four-dimensional dynamics that we have found is that of a self-dual gravity (the supersymmetric extension of the theory can be found by simply extending the current algebra). Introducing

$$\begin{aligned} J_0 &= J_+ + J_-, \\ J_1 &= J_+ - J_-, \end{aligned}$$

one has the algebra

$$\begin{aligned} \left[J_0^\alpha, J_0^\beta \right] &= f_{\alpha\beta\gamma}\, J_0^\gamma, \\ \left[J_0^\alpha, J_1^\beta \right] &= f_{\alpha\beta\gamma}\, J_1^\gamma - \delta_{\alpha\beta}\delta', \\ \left[J_1^\alpha, J_1^\beta \right] &= 0, \end{aligned}$$

and the equations of motion

$$\partial_t\, J_0 + \partial_x\, J_1 = 0, \tag{3}$$

$$\partial_t\, J_1 - \partial_x\, J_0 + \{J_0, J_1\} = 0, \tag{4}$$

where we have the Poisson bracket and the four-dimensional notation. Solving the first equation with

$$\begin{aligned} J_0 &= \partial_x\, \Omega\, (t, x, \sigma_1, \sigma_2)\,, \\ J_1 &= -\partial_t\, \Omega\, (t, x, \sigma_1, \sigma_2)\,, \end{aligned}$$

one has for the second:

$$\left(\partial_t^2 - \partial_x^2 \right) \Omega + \epsilon^{rs} \partial_r \left(\partial_t \Omega \right) \partial_s \left(\partial_x \Omega \right) = 0.$$

This equation of motion is associated [14, 15] with the (Plebanski) action

$$\mathcal{L} = \int dt\, dx\, d\sigma_1\, d\sigma_2 \left(\frac{1}{2} (\partial_t \Omega)^2 - \frac{1}{2} (\partial_x \Omega)^2 + \frac{1}{3} \Omega \, \{\partial_t \Omega, \partial_x \Omega\} \right).$$

We now make the following observation: in the mechanism which produces self-dual gravity, we recognize elements of non-abelian duality. That is done in the original works [13, 15] as a purely classical transformation. It is known that at the quantum level the correct procedure for this is as in Refs. [16-19]. Consequently, the Plebanski theory is only equivalent at the classical level and the full theory of quantized self-dual gravity is to be more precisely specified.

Let us then summarize the basic features of the present theory. The Hamiltonian

$$H = \int \left(\frac{\alpha^3}{6} + \alpha \, J\,(x,\sigma)^2 \right) + c \int \frac{J(x,\sigma)J(y,\sigma)}{(x-y)^2} + \gamma \, W_0(J) + V$$

describes:

1. a scalar field α representing a dilaton,
2. a four-dimensional field $J(x, t, \sigma_1, \sigma_2)$ originating from the $SU(\infty)$ current algebra and representing a metric of 4-dimensional self-dual gravity,
3. an additional current-current interaction which we interpreted as coming from matter (this explains the arbitrary coefficient c present).

We have found that, starting with the light cone quantum membrane, a four-dimensional world volume structure arises. The precise form of 4-dimensional dilatonic self-dual gravity with matter is based on the **non-abelian** dualization (with the corresponding Susy extension) [20]. The quantum theory, however, is defined by the operator Hamiltonian presented in terms of an $SU(\infty)$ Kac-Moody algebra. This then offers a framework for studying the quantum spectrum.

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